Mars seasons: Development of when $L_S = 0$ and functions for $L_S(t)$

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0.1 Introduction

This document retains only the final results of the development of L_S relations for the 2015 Icarus paper led by Sylvain Piqueux, Piqueux15=[4], referred to here as Paper 1. The development took place over 2014may24 to 2015jan25.

The top IDL program is **QLSUBS**, written as a large case statement with actions labeled with integers, many actions are residual from this or earlier developments and were not used in the final calculations.

All calculations are done in "ephemeris time" (Barycentric Dynamical Time, TDB) the native time system of the JPL DE 430 planetary ephemeris. The working time base is epoch J2000.0 (2000 January 1, 12 hour) and times relative to this are called 'MJD'

The analytic expressions developed are empirical and are based on the 536 year period from 1607 to 2143 (285 Mars years)

0.1.1 Notation

Program and routine names are shown as **PROGRM**. Code variable names are shown as `variab` and within equations as `variab`. Input parameters are shown as INPUT and within equations as INPUT. File names are shown as *file*.

Actions within a case statement are indicated by "@" followed by some integer. These appear mostly in order in the main program, but the integers have no meaning in terms of execution sequence.

Some residuals are given as the Mean Absolute Residual, MAR, others as standard deviation, StdDev.

1 Basic relations

L_S is the planetocentric longitude of the Sun; i.e, the longitude of the Sun in a coordinate system centered on the planet and defined by the +Z axis along the normal to the planets orbit plane and the +X axis toward the planet's vernal equinox.

The Vernal Equinox (VE) is toward the intersection of the body equator and the body orbit at which the Sun rises into the "North" hemisphere; it is from the planet to the Sun. Thus VE is along spinAxis-cross-OrbitNormal. Near this time, the MH vector (heliocentric Mars) is descending below the planets equatorial plane.

A consistent coordinate system must be used. DE430 uses the International Celestial Reference System (ICRS) and Barycentric Dynamical Time (TDB), and that is used through out here. The Mars pole of Kuchynka14=[3] also uses the ICRS but uses Terrestrial Time (TT), which differs from TDB by a periodic function of less than 2 milli-second; this difference is ignored here.

1.1 Philosophic Descisions

When working at high accuracy, seemingly similar definitions become distinct.

I. How to define the pole direction of a planet when the most accurate definition includes periods less than the planets year? The four options coded are described in §3. Kuchynka 2014 without the high-order terms was chosen.

II. How to define a planet's orbital plane when the most accurate definition of the planet's position over time is a numerical ephemeris?
Options for orbit normal. @12:16 Second digit of VE code
 1=a) instantaneous cross product of position and velocity; @444 mbx3
 2=b) each MY fit to positions throughout that year @444 pfit
 3=c) Standish Keplerian elements. Access @21
 4=d) secular trend of (b) @45 pcc and cnorr
Expect c and d to be similar. The results for each of these are shown in Figure 1

III. How to define the instant of spring; $L_S = 0$:
 y) when the longitude relative to the VE increases past zero?
 z) when the sun rises above the planets equator?
As the obliquity approaches zero, the latter could have large variations due to perturbations out of the mean orbital plane. However, with obliquity near 26°, the sensitivity to out-of-plane perturbations is only about a factor of 2 greater than perturbations in longitude. Software has the option to do either.

Due to planetary perturbations, for Mars definition z yield times for $L_S = 0$ that have a average difference from definition y by 0.00035 days, with a range of -0.00093 to 0.00084 days.

IV. Because the Vernal Equinox is moving in inertial space (the ICRS), need to decide whether and how to set it for a given Mars year. e.g.:
 i) Reevaluate at every date

ii) Fix at the start of each Mars year.
iii) Fix at the middle of each Mars year.

The numerical results here use, unless explicitly stated otherwise: I, Kuchynka 2014 without the high-order terms; II, option d; III, option y; IV, option ii.

2 Access to planetary positions

JPL ephemeris DE 430 is accessed in IDL through a C wrapper to call **dpephem.f** which then calls the JPL Fortran software; the interface can run DE430 in the heliocentric or Barycentric (native), A.U. or km (native) modes; here it is set for heliocentric, km mode.

2.1 Defining the orbit plane

@44: Find the heliocentric position of Mars at uniform time intervals of 1/36 of a MY over the entire period, phased to avoid times near $L_S = 0$ that could yield values of ϵ or $360 - \epsilon$. Save the time, the position and velocity vectors.

@444: For each MY, fit a plane to the positions, convert to the direction of the normal to this plane (the annual angular-velocity vector). Store in `pfit`

@45. Compute the secular trend to orbit normal (angular velocity vector) several ways and select one result.
for polar co-latitude p (90.-declination) and polar azimuth (Right Ascension) independently:
Using the angular-velocity vector direction, either all the individual times or the average for each MY
Fit linear or quadratic in time.
Select one result; quadratic in time to all the individual times. Store in `cnorr`

Orbit normal in ICRF: where T is Julian centuries from epoch J2000:
RA= $273.^\circ 373218337 - 0.029859329 66T - 4.829810557 \times 10^{-5}T^2$
Dec= $65.^\circ 322934512 - 0.00128897471T + 4.460153556 \times 10^{-5}T^2$

3 Access to spin-axis direction

The results of Folkner97=[2] and Kuchynka14=[3] are coded in **marspin.pro** .
Options for spin axis: 4 codes in MARSPIN. @12:6 First digit of VE code
1= Folkner 1997 linear
2= Kuchynka 2014 without the high-order terms
3= Kuchynka with all terms
4= Low-order fit to Kuchynka; 15555 random times over 0:4000 CE
Linear in RA and quadratic in Declination was found to be adequate.

Option 2 was used: the determination of [3] omitting terms with a period of one Mars year or less, all $< 0.00024^\circ$.
RA= $317.^\circ 269202 - 0.10927547\ T + 0.419057 \sin(79.398797 + 0.5042615\ T)$
Dec=$54.^\circ 432516 - 0.05827105\ T + 1.591274 \cos(166.325722 + 0.5042615\ T)$

4 Finding the time of L_s=0

Done @4. There were some early reruns of some actions to refine times and trends.

$L_S = 0$ is found by advancing the time by one tropical year (686.971 days) after the prior solution and then a Newtonian iteration on the residual adjusting the time and recomputing the pole direction and DE430 planetary position for each iteration until the tolerance was met.
Get the RA and Dec. of the spin axis; convert to a Cartesian unit vector `zmaxu`.
Get the Heliocentric Mars vector using DE430; convert to a unit vector `mhaxu`
Get the RA and Dec. of the orbit normal; convert to a Cartesian vector `zon`
Compute vector to Vernal equinox: `VEaxx`= zmaxu × zon; convert to unit vector

Generate Y axis of Mars Orbital system **O** which has Z along the orbit normal and X toward the anti-VE. Then generate rotation matrix from ICRF to Orbital systems. Rotate heliocentric Mars position unit vector into the Orbital system; error is the Y component.

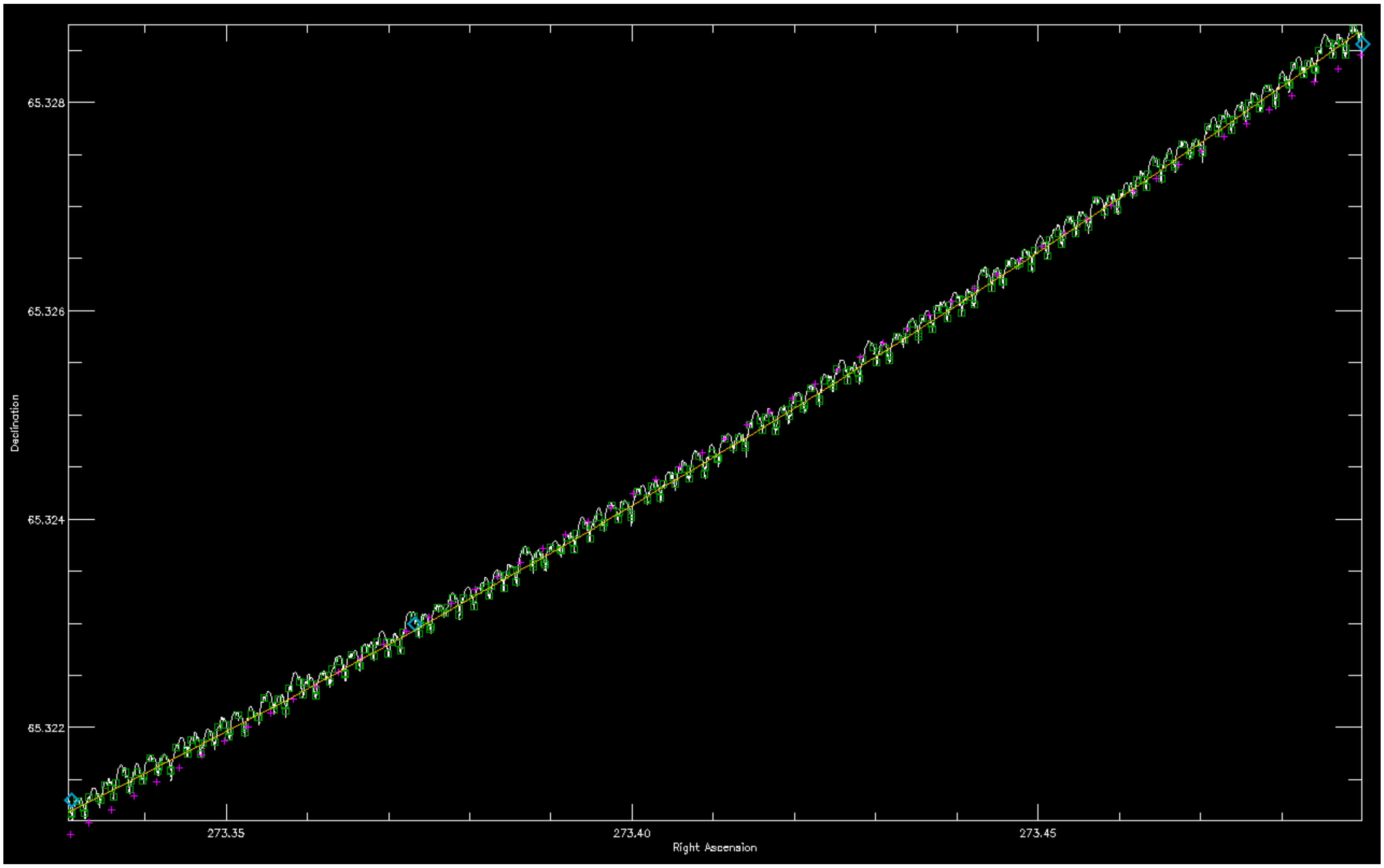

Figure 1: The location of the orbit normal over time. Abscissa is the Right Ascension and the ordinate is the Declination, both in radians in J2000 equatorial coordinates. White lines are the trace of the instantaneous cross-product of heliocentric Mars position and velocity for 36 times each Mars year from MY -184 to MY 100, time increases toward the left. Green squares are the normal to the best-fit plane to the heliocentric positions for each Mars Year. The pink plus signs are the results of a linear fit as a function of time to the cross-product data, shown every 5 MY from the start (upper right) to the end (lower-left) of this period. The yellow line is the fit quadratic in time.The blue diamonds are values derived from the secular Keplerian elements of [5] (valid over 1800-2050 AD) at the ends of this period and at the epoch J2000

Tolerance used was 1.E-9 radians. Save the time and the MH vector. Compute and save the duration of each year (subtract successive MY start times)

4.1 Results

The StdDev of the duration of Mars years is 0.01717 days, the total variation is 0.07252 days.

From these dates, develop a fit to the time of $L_S = 0$ over the total period being considered. t=cmy$_0$+cmy$_1$*MY [+ cmy$_2$ *MY^2].

Linear fit to time of LS=0 using integer MY's from 2000. as independent variable:
    `cmy`= 151.26228, 686.97078 . Residual mean and StdDev= 4.3335727e-11, 0.014038043
Trying quadratic fit
    `cmp`= 151.26034, 686.97089, 8.5198488e-07 . Residual mean and StdDev= 1.2219001e-10, 0.013052554
Little improvement, StdDev reduction is proportionally small

4.2 The output table of MY

The start and end of the 1.5 page table is shown in Table 1

Table 1: Starting date of Mars Years, as ephemeris days from J2000.0, JD 2451545. Central part omitted, full version in Paper 1

Mars	Earth			Days from	Mars	Earth			Days from	Mars	Earth			Days from
Year	year	mon	day	J2000.0	Year	year	mon	day	J2000.0	Year	year	mon	day	J2000.0
-184	1607	4	25	-143425.628	-134	1701	5	11	-109077.093	-84	1795	5	26	-74728.549
-183	1609	3	12	-142738.649	-133	1703	3	29	-108390.130	-83	1797	4	12	-74041.596
-182	1611	1	28	-142051.672	-132	1705	2	13	-107703.136	-82	1799	2	28	-73354.610
-181	1612	12	15	-141364.723	-131	1707	1	1	-107016.185	-81	1801	1	16	-72667.641
-180	1614	11	2	-140677.738	-130	1708	11	18	-106329.219	-80	1802	12	4	-71980.693
6	1964	9	5	-12901.184	51	2049	4	26	18012.511	96	2133	12	15	48926.192
7	1966	7	24	-12214.213	52	2051	3	13	18699.451	97	2135	11	2	49613.155
8	1968	6	10	-11527.271	53	2053	1	28	19386.438	98	2137	9	19	50300.150
9	1970	4	28	-10840.292	54	2054	12	16	20073.435	99	2139	8	7	50987.124
10	1972	3	15	-10153.297	55	2056	11	2	20760.397	100	2141	6	24	51674.083

5 Develop an L_s dataset

@433: Begin with the uniformly-spaced times described at the start of §2.1. At the beginning of each year:
- Define the spin axis unit vector `zmaxu`(4 options, use the same as identified in §1.1)
- Define the orbit normal vector zon (4 options, use the same as identified in §1.1)
- Compute the anti vernal-equinox unit vector: `EVaxu`
- Form the rotation matrix from ICRF to Mars orbital
- For each season

  Get the heliocentric mars unit vector `MHaxu`

  Take arc-cos of the dot product of EV and MH to get the longitude relative to the VE, this is L_S (radians).

  Save the fraction of the current Mars year, L_S, and the longitude in the Mars orbital system (the true anomaly): array `xxyy`.

Using a plane fit to the orbit for each MY and the spin-axis at the start of each MY, the obliquity is found to be 25.191846° +0.012085 T; the MAR is 0.00009 °. In comparison, Folkner97=[2] gives an obliquity [at J2000] of 25.189417 (35) and a rate of 1 (16) milli-arc sec / year where the numbers in parentheses indicate uncertainties in the final digit or digits; the rate values convert to 2.8e-5 and 4.4e-4 degrees/century. Their work used the mean Mars orbit of 1980; using $T = -.2$ in the above equation yields an obliquity of 25.189429°, in agreement with Folkner97=[2].

6 Fitting analytic relations: FITMYLS

All analytic expressions for Ls are empirical derived from fitting 285 Mars Years (MY) from 1607 to 2141. They are based on fits to minimize the root-mean-square (RMS) residual at 36 times per Mars year uniformly spaced over the entire interval and use a form similar to [1] [equation numbers in brackets indicate similar equations in that work]. All calculations are done in double-precision.

@48: Subroutine to fit Ls (degrees) as function of date. Has several options for which terms are included:
Linear terms: solved for using Singular Value Decomposition

  up to 3 of: a_0 constant, a_1 linear in t, a_2 quadratic in T

  the amplitudes for up to N planetary perturbation terms of the form: $A_i \cos\left[2\pi\frac{t}{\tau_i} + \frac{\pi}{180}\phi_i\right]$

  Sine series in mean anomaly M: $\sum_{k=1}^{N} s_k \sin(k * M)$

Parameters: solved for using N-dimensional successive approximation

  up to 2 of: m_0 mean anomaly offset, m_1 mean anomaly rate

  1 or 2 for equation of center: e_0 eccentricity constant, e_1 eccentricity linear rate

  the phases ϕ_i for up to N planetary perturbation terms.

Or, up to N planetary perturbation terms can be included with pre-determined amplitudes, periods and phases.

May not do both sine series and equation of center at the same time.

If only one parameter, use Brent's method for the non-linear part, if more then use the simplex algorithm. I have recoded both of these to be "called from above" rather than have them call a function to evaluate the error; **brentx.pro, amoebax.pro** .

The simplex method allows range limits for each parameter.

Because wish to treat tropical year terms as linear coefficients, but need them before the fit, must have an initial guess that is good enough that no angle wraps across 0=360. Then, can refine the annual ramp with a linear process.

To improve the stability of the non-linear algorithms (the simplex algorithm), the parameters for mean anomaly and eccentricity are treated as offsets from the initial guess.

Don't worry about large angles, trig. functions take modulo well. Tested to be at the limit of the numerical precision.

The interface arguments are defined in the routine internal documentation. The returned function is a vector of:
  the number of iterations; followed by
  the final (minimum) error; followed by
  the values of the found non-linear parameters; followed by
  the values of the found coefficients of linear terms.
In addition, an array of the history of convergence is returned.

Sequence:
Form the basis functions which are invarient.
Set parameters at initial guess
LOOP:
  Form the basis functions which depend upon parameters
  Form the linear part of the tropical year: `ytrop`= $a_o + a_1 t$; subtract from L_S vector
  Get least-squares solution for coefficients: call to FITSVD or SVDESIGN
  Form error value [check/correct for any values near 360]
  Update the parameters: call to AMOEBAX or BRENTX
    If have not reached solution, repeat LOOP
Add back `ytrop` to get the model vector.

Control parameters input to **FITMYLS**, with typical values. @13 in the main program can set values of the longer set `parj` from which values at the listed index are extracted to compose `pam` input to **FITMYLS**

```
parj pam val
 6   0]= 600 Max iterations. 0=evaluate function at p4[0,*]
 7   1]=  45 -10*log_10 fractional tolerance
 9   2]=  40 -10*log_10 adequate MAR
10   3]=  3  Number of basic terms
 5   4]=  4  Nums for sine Must be 1+
 8   5]=  0  Deg for Cos NOPE
14   6]=  1  Flag: not 0 means Treat year length as a parameter
11   7]=  7  Number of PDS terms, may be 0; each involves a parameter
                 -N: use preset values   +N: fit here
12   8]=  0  Flag. Set to use normal equations, else uses design matrix
13   9]=  2  Number of eccentricity coefficients, le 0 means use sine series
```

The seven largest PCPs were assumed to be those listed in Allison00=[1] and their periods τ_i were kept fixed at the values computed from the planetary orbit periods. ; the phase and amplitudes were fit in a run of FITMYLS that had 11 free non-linear parameters and 10 free linear coefficients at one time for a total of 21 values. The parameters were: m_0, m_1, e_0, e_1, and ϕ_1 thorough ϕ_7 ; the coefficients were a_0, a_1, a_2, and A_1 thorough A_7; see Table 2 of Paper 1. The residuals of this fit was then processed [see next Section] to derive successively fits for the next 10 PCPs in which all three values were free for each fit, for an additional 30 values.

6.1 Assessment of planetary perturbations: FFTLOOK

Once a fairly good fit has been found, the residuals may be examined for periodic functions expected for planetary perturbations. Because the 10260 dates used are uniformly spaced, the residuals from any fit can be processed by a Fast Fourier Transform. Because the total range of the form being fit is only ± 0.007°, single precision was considered adequate.

The commensurate periods for planetary perturbations (PCP) can be computed from the orbital periods of each planet, but phase and amplitudes are (here, at least) found emperically. Planets from Venus to Saturn were considered; potential magnitude, based on scaling the gravitational attraction at mean closest approach, is a factor of 30 or more smaller for other planets. Period factors up to 5 were considered.

Step 1: Compute all reasonable commensurate periods:
- Read the file of Standish orbit elements
- Compute the orbital period for each plant based on its semi-major axis

- For each planet, skipping Mars: pari[14] to pari[15]
– For factor N of 1 to pari[13] for the planet
— For factors M of 1 to pari[2] for Mars
—- Compute the commensurate period: $\tau = 1/(N/\tau_{planet} - M/\tau_{Mars})$
- Sort the commensurate periods into increasing values

@116: Setup for commensurate, The following actions:
2.... Use input waveform
22... Plot waveform
67... Calc commensurate periods
-1... pause
4.... FFT Req 21
52... Make aaa based on pari[2:3] REQ 4
12... Modify pari
52... Make aaa based on pari[2:3] REQ 4
54... Find modulus peaks

@142: then 124. Loop over commensurates
2.... use input
22... Plot waveform
4.... FFT Req 21
41... Combine several actions for peak loop REQ 4
Form the modulus and find the maximum
Convert to period in days and find the closest commensurate period
58... Trials to localize phase
For ntry=pari[9] uniformly spaced phase angles within 0:360
Compute a waveform of 1/2 the expected amplitude at the estimated period.
Subtract it from the target waveform, compute the MAR
Find the index of the minimum and do a parabolic fit to 3 points centered on this
Compute the location of the minimum
59... Find best phase REQ 58 immediate before
Treat only phase ϕ or phase and period τ as parameters and iterate to find the minimum least-squares solution treating amplitude as linear parameter. Because there is only one linear coefficient, it can be found as

$$c = \frac{\sum_i W_i F_i}{\sum_i F_i^2} \tag{1}$$

where W is the target waveform and $F = \cos(2\pi/\tau + \phi)$ is the commensurate period form
592.. CHART eee
595.. Apply the latest solution: subtract it from the wave being fit
If not reached the count of FT peaks to treat, back to the top of this loop

The low-frequency section of the FT of the residual of a best 7-PCP fit is shown in Figure 2. The results of a parabolic fit the the highest 10 peaks are shown in Table 2

6.1.1 Control parameters:

```
@12: Integer values: pari
      0       1  i1: first \ FT for Plot
      1     900    " number/ @52,62,72
      2       1  j1: first \ Varib Loop
      3     900    " number/ keep
      4      62  k1: first \ sum @46
      5       7    " number/ Plot
      6       1  Print actions
      7       0  Flag: Varib is phase, 0=freq
      8       0  Flag 0=cos 1=sin
      9       9  Ntry for phase
     10      40  MaxIter
     11       1  0=FITSVD 1=SVDESIGN
     12       5  @67 * for Mars
     13       5   " for others
     14       2  First planet 2=V
```

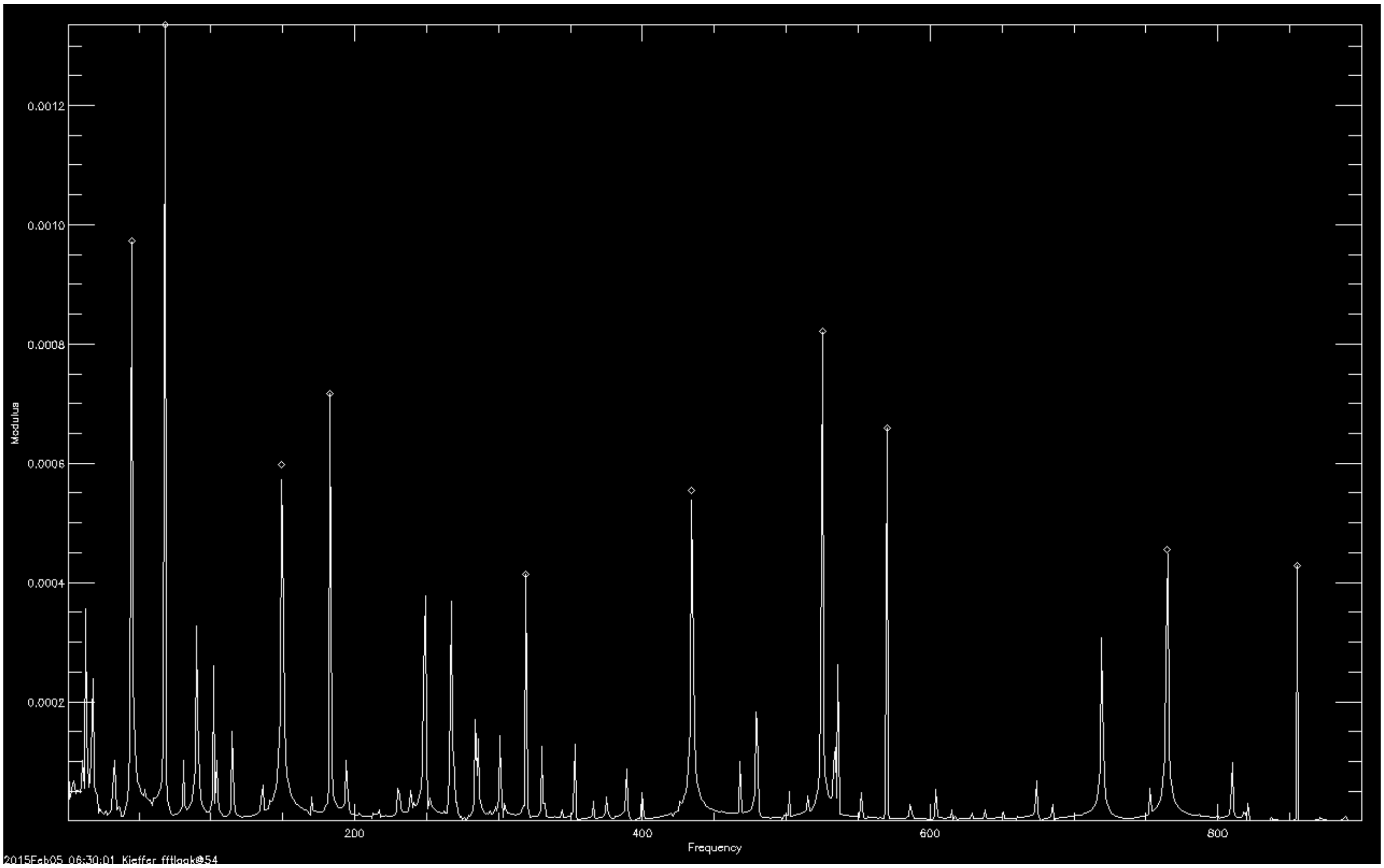

Figure 2: Low-frequency end of the modulus of a Fourier transform of the residuals after the 7-planet fit. Diamonds show the location and maximum value of a parabolic fit to the 3 points centered on the highest 10 peaks in the modulus.

```
      15       6  Last " 6=S
      16      10  Number of peaks to retrieve
      17       4  Number pcp
      18       2  Num basis func. 1 or 2

@16 Float values: parr
       0      1024.00  NumPoints INT
       1     -7.00000  spare
       2      1.00000  1: amplitude
       3      32.0000   " period
       4      0.00000   " phase: rad
       5      0.00000   " bias
       6      0.00000  2: amplitude
       7      100.000   " period
       8      0.00000   " phase: rad
       9      0.00000   " bias
      10      3.00000  Y plot min
      11     0.600000   " max <min=auto
      12      11.0000  Num Varib/Freq INT
      13     0.314159  Phase delta
      14    0.0500000  Freq Delta
      15  1.00000e-12  Tiny
      16  1.00000e-05  @59 fit tolerance
      17  1.00000e-07  " adequately small error
      18  0.000600000  yplot@68
      19      1.00000  Phase margin @59, rad
```

Table 2: Planetary commensurate perturbations matched to the top 10 spikes in the Fourier Transform (FT) as fit with a parabolic function. Maxima listed as 1000. times their true value in degrees. The second listing for the ninth PCP is a close match to a period not automatically generated

i count	FT peak index	Fit center	FT max.	parab. fit max.	parab. fit period	PCP period	PCP item
8	67	67.99	1.335	1.335	2879.67	2882.65	4M-2E
9	44	45.03	0.971	0.972	4347.62	3309.58	M-5J
9						4323.98	(2M-E)*3/4
10	524	524.98	0.820	0.821	372.94	373.06	2M-J
11	182	183.01	0.717	0.717	1069.81	1069.21	5M-3E
12	569	570.00	0.660	0.660	343.49	333.92	M-V
13	148	149.32	0.573	0.597	1311.19	1309.69	M-3J
14	433	434.24	0.539	0.554	450.87	450.62	2M-3J
15	764	764.83	0.448	0.455	255.99	256.05	3M-2J
16	854	855.00	0.428	0.428	228.99	233.98	3M-S
17	318	319.00	0.415	0.415	613.76	613.85	3M-E

6.2 With no planetary perturbation terms

A simple relation is within 0.05° over the entire period:

$$M = 19.^\circ 38095 + 0.524020769\ t \tag{2}$$

$$L_S \sim 270.^\circ 38859 + 0.524038542\ t + 10.67848 \sin(M) + 0.62077 \sin(2M) + 0.05031 \sin(3M) \tag{3}$$

6.3 With the largest 7 planetary perturbation terms

The linear-rate angle based on the tropical year is α where t is the ephemeris time from J2000.0 in days and T the time from J2000.0 in Julian centuries (36525 days).

$$\alpha = a_0 + a_1\ t + a_2\ T^2 \simeq 270.^\circ 389001822 + 0.52403850205\ t - 0.000565452\ T^2 \qquad [17] \tag{4}$$

$$M = m_0 + m_1\ t \simeq 19.^\circ 38028331517 + 0.52402076345\ t \qquad [16] \tag{5}$$

where M is the mean anomaly. Using eccentricity

$$e = e_0 + e_1 T \simeq 0.093402202 + 0.000091406\ T \tag{6}$$

to evaluate the true anomaly ν using the equation of center:

$$\begin{aligned} \Delta \equiv \nu - M \quad &= \left(2e - \tfrac{1}{4}e^3 + \tfrac{5}{96}e^5\right)\sin(M) \\ \text{in} \quad &+ \left(\tfrac{5}{4}e^2 - \tfrac{11}{24}e^4 + \tfrac{17}{192}e^6\right)\sin(2M) + \left(\tfrac{13}{12}e^3 - \tfrac{43}{64}e^5\right)\sin(3M) \qquad [4] \\ \text{radians} \quad &+ \left(\tfrac{103}{96}e^4 - \tfrac{451}{480}e^6\right)\sin(4M) + \tfrac{1097}{960}e^5\sin(5M) + \tfrac{1223}{960}e^5\sin(6M) + \mathcal{O}(e^7) \end{aligned} \tag{7}$$

Major perturbations by other planets are treated as:

$$\text{PCPs} = \sum_{i=1}^{N} A_i \cos\left[2\pi\frac{t}{\tau_i} + \frac{\pi}{180}\phi_i\right] \qquad [18] \tag{8}$$

where τ is the commensurate period in days and ϕ is in degrees; coefficient values are given in Table 3

$$L_S = \alpha + \frac{180}{\pi}\Delta + \text{PCPs} \tag{9}$$

This model is adequate to match the DE430 calculations with maximum error of 0.0073° over the 536 years of MY-184 through MY+100; the RMS residual is 0.00207°.

These residuals are shown in Figures 3 and 4.

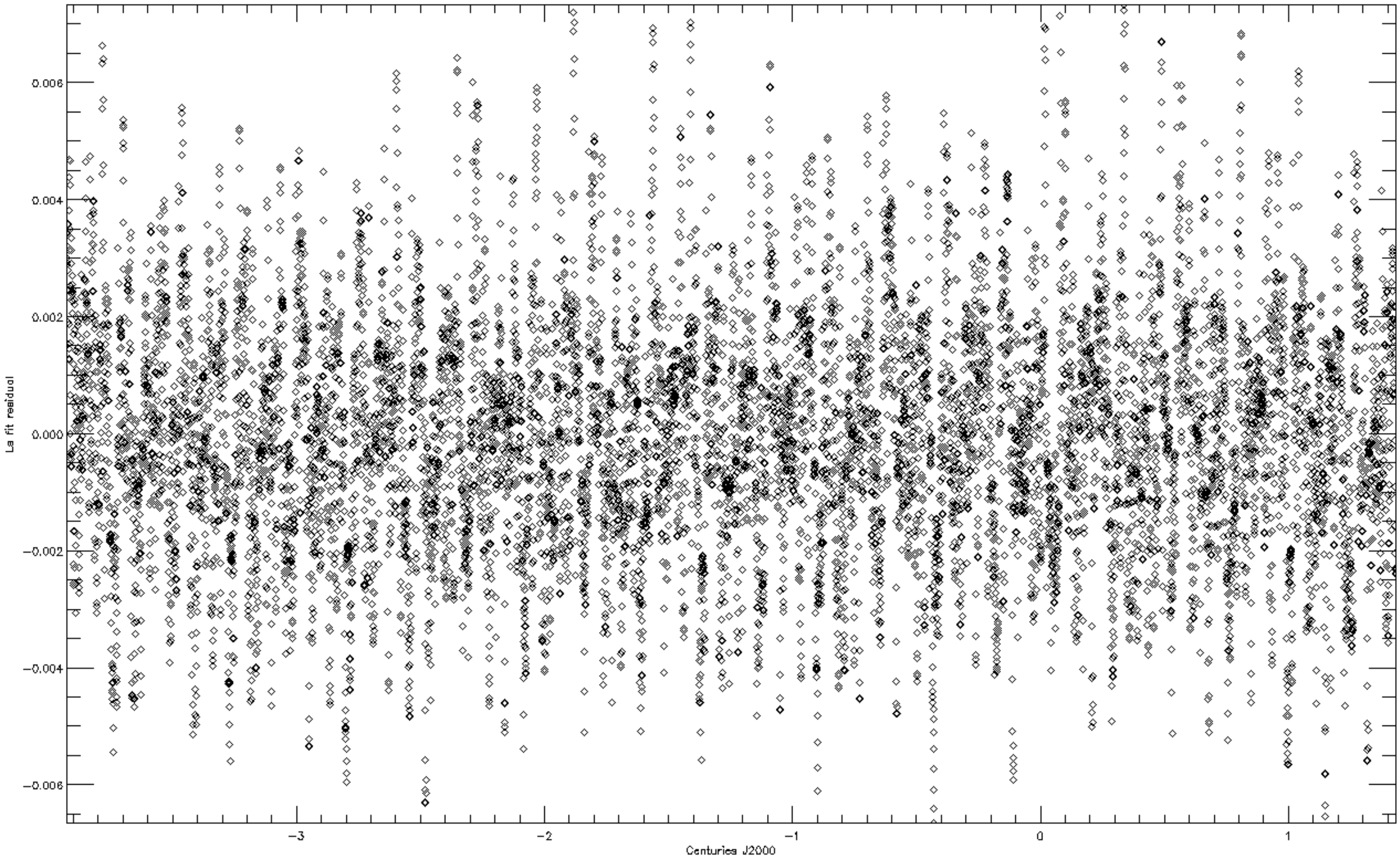

Figure 3: Residuals from the fit with seven PCPs; all 10260 points

6.4 Additional planetary perturbation terms

— Starting with the residuals in the prior section, the methods described in §6.1 were applied to calculate nine additional PCPs, allowing the period, phase and amplitude to be free. The resulting values are listed in Table 3.

With all terms in Table 3, the maximum error is 0.0045°and the RMS residual is 0.00105°.

-

6.5 Testing

The three relations above were coded as an IDL routine **LsHK** with one optional keyword to select between them.

7 Major Arrays

```
nmy=long(pari[5])           ; Number of full MY to do
nls=pari[15]                ; seasons/year
ntot=nmy*nls                ; total number of times
ndata Number used in fit @48, usually same as ntot

array dim    make read items
mba  [nmy,8]  4 402  MJD error slope N_iter MHauX MHauY MHauZ LenYear
                  Conditions at the start of each MY. Unit vector.

mbx3 [ntot,7]  44 442  MJD Xpos Ypos Zpos Xvel Yvel Zvel
                  Heliocentric Mars at uniform times over the entire period, in km and km/s

mopp [ntot,8] 444 446  XoNormu YoNormu ZoNormu XoNormv YoNormv ZoNormv oNormp oNormq
```

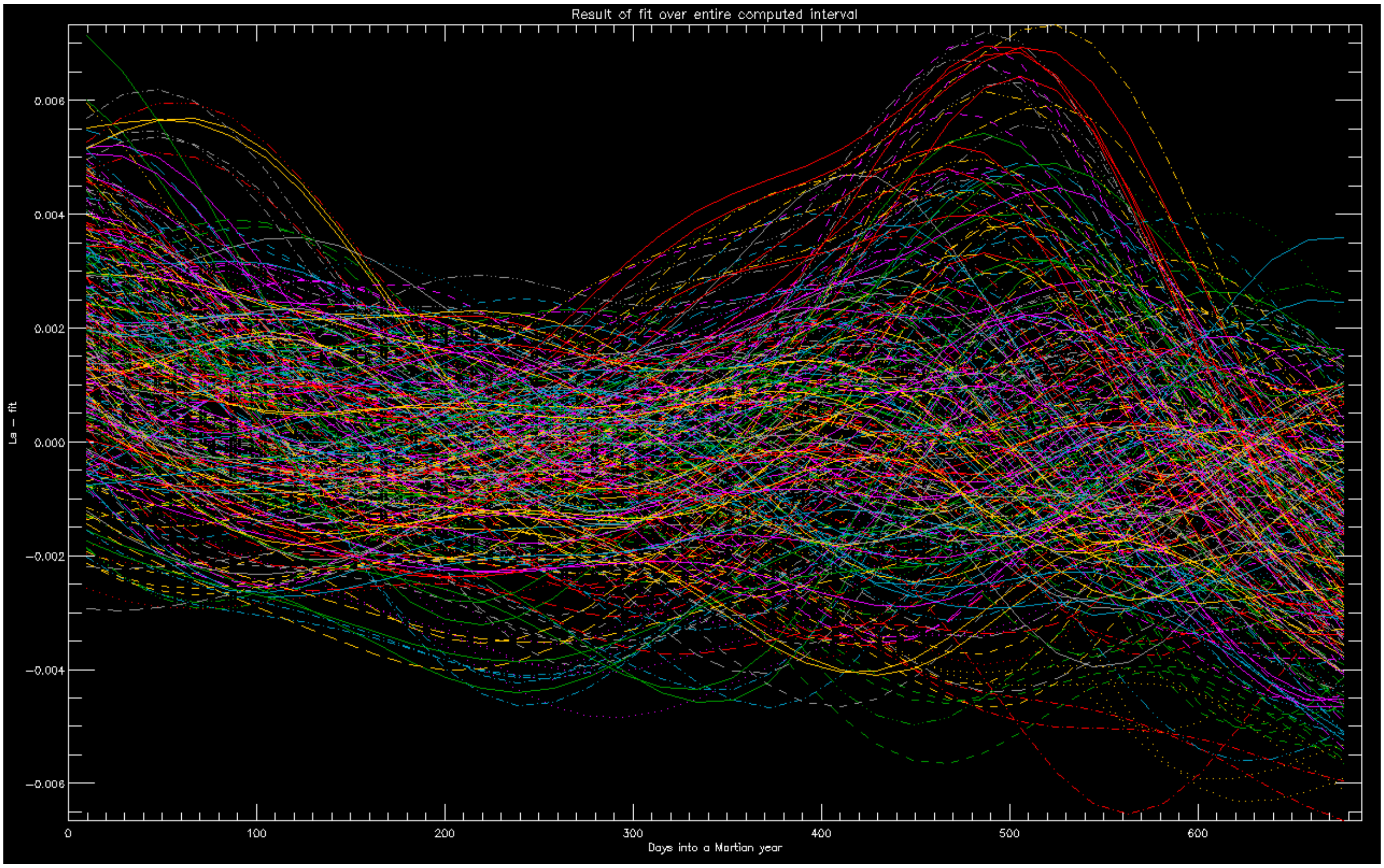

Figure 4: Residuals from the fit with seven PCPs versus L_s

```
                  First 3 are unit vector toward instantaneous angular velocity
                  Second 3 are un-normalized (km^2/sec)
                  Last 2 are polar angle and azimuth in radians

pfit [nmy,6]  444 446  c0 c1=x c2=y resid Normp Normq
                  Each year fit a plane to Mars positions
                  Last 2 are polar angle and azimuth in radians

xxyy [ntot,3] 433 470  FracYear Ls LsP
                  With each year, fractional time into that year,
                  Ls in degrees based on angle from VE
                  Ls in radians based on rotation of heliocentric vector into Orbital system

All above are Held with leading 'h' @467

Fit input and output .sav file name set @492, save @493, restore @494

ccf yfit p4 are Held with leading 'h' @487



Look at effect of short-period terms in MARSPIN
tcen=mbz/julcen
pp2=MARSPIN(tcen,kin=2) ; without high-order
p3=MARSPIN(tcen,kin=3)
qq=pp3-pp2 & chart,qq[*,0:1] ; nearly linear
xa=min(qq[*,0],max=xb) ; RA
ya=min(qq[*,1],max=yb) ; Dec
```

Table 3: Planetary perturbation terms. 2E-3M is shorthand for $1/(E/2 - M/3)$ where E is the length of a year for Earth, M for Mars, J for Jupiter and V for Venus. Years are based on the semi-major axes at epoch J2000 given by [5]; period in days was held to match this for the first 7 rows, and allowed to vary slightly to minimize the RMS residual for the last 9.

Planets	Commensurate Period years	days τ_i	Amplitude milli-deg $1000A_i$	Phase Degree ϕ_i
M-J	2.23511	816.3755210	7.0591	48.48944
M-2J	2.75373	1005.8002614	6.0890	167.55418
2M-2J	1.11756	408.1877605	4.4462	188.35480
2M-E	15.78456	5765.3098103	3.8947	19.97295
E-M	2.13533	779.9286472	2.4328	12.03224
2E-3M	2.46939	901.9431281	2.0400	95.98253
V-3M	32.80201	11980.9332471	1.7746	49.00256
4M-2E	7.89228	2882.1147	1.34607	288.7737
(2M-E)*3/4	9.06113	4332.2204	1.03438	37.9378
2M-J	1.02138	373.07883	0.88180	65.3160
5M-3E	2.92735	1069.3231	0.72350	175.4911
M-V	0.91423	343.49194	0.65555	98.8644
M-3J	3.58573	1309.9410	0.81460	186.2253
2M-3J	1.23373	450.69255	0.74578	202.9323
3M-2J	0.70103	256.06036	0.58359	212.1853
3M-S	0.64060	228.99145	0.42864	32.1227

```
print,xa,xb,ya,yb   6.7146979e-05   6.9633770e-05  -0.00016221086  -0.00015811039
print,(xb-xa)/xb,(yb-ya)/yb   0.035712426    -0.025934238
Thus: small and virtually constant

qa=mba-hmba ; 36a3-36U
 za=min(qa[*,0],max=zb)
print,za,zb,(zb-za)/zb   -0.00038100556  -0.00037456659 -0.017190449
```

References

[1] M. Allison and M. McEwen. A post-Pathfinder evaluation of areocentric solar coordinates with improved timing recipes for Mars seasonal/diurnal climate studies. *Plan. Space Sci.*, 48:215–235, February 2000. See `http://www.giss.nasa.gov/tools/mars24/help/algorithm.html`.

[2] W. M. Folkner, C. F. Yoder, D. N. Yuan, E. M. Standish, and R. A. Preston. Interior Structure and Seasonal Mass Redistribution of Mars from Radio Tracking of Mars Pathfinder. *Science*, 278:1749–+, December 1997.

[3] P. Kuchynka, W. M. Folkner, A. S. Konopliv, T. J. Parker, R. S. Park, S. L. Maistre, and V. Dehant. New constraints on Mars rotation determined from radiometric tracking of the Opportunity Mars Exploration Rover. *Icarus*, 229:340–347, 2014.

[4] S. Piqueux, S. Byrne, T.N. Titus, C.J. Hansen, and H.H. Kieffer. Enumeration of Mars Years and Seasons since the Beginning of Telescopic Exploration. *Icarus*, in print:xx, 2015.

[5] E.M. Standish and J.G. Williams. Keplerian elements of the approximate positions of the major planets. In *Explanatory supplement to the Astronomical Almanac*. online at `http://iau-comm4.jpl.nasa.gov/keplerformulae/kepform.pdf`, 2006. Section 8.10.

A Other $\mathrm{L_s}$ software

A.1 NAIF CHRONOS

```
Email from Chuck Acton 2015may23
Hugh,

There is a special version of Boris' CHRONOS time conversion web program
that can compute L-sub-s.

   http://naif.jpl.nasa.gov/cgi-bin/chronos_phx.pl?setup=phxtime

You should be able to try it now.

  Using the "To" selector drop-down select LST.
  Using the adjacent selector drop-down select LSUN.

Note: it outputs in the range of -180 to +180, rather than 0 to 360.

Also, it works only for the time span of the loaded planet ephemeris file.
The current planet ephemeris available to PHX CHRONOS does not go way out
to 2141.  But if you would like him to do so, he can load the DE431
ephemeris into PHX CHRONOS so you could make calculations out that far.
--------------
My trials: e.g., 2451545.0JD ET  SCET LST LSUN
PHX_NOM_SCLK: 2451545.0   JD (ET/SCET) --- -85.629 (LST/LSUN)
Find that its date limits are approximately:
PHX_NOM_SCLK: 2449719.JD (ET/SCET) --- 39.446 (LST/LSUN)
PHX_NOM_SCLK: 2458119.JD (ET/SCET) --- 108.535 (LST/LSUN)
These are  -1826 and +6574 days from J2000.0
```

A.2 SPICE code

The SPICE Fortran version lspcn.f accommodates aberrations; it is based on the instantaneous angular momentum vector (normal to the instantaneous orbit plane) and instantaneous plane spin axis, based on the ephemeris and rotational elements files that are currently loaded in the SPICE system.

A.3 Laboratoire de Meteorologie Dynamique (LMD)

LMD has a web page for conversion of Earth date and time to L_S with 0.1°resolution.
http://www-mars.lmd.jussieu.fr/mars/time/martian_time.html
For the start of MY 32 yields it 359.9 [limit of output precision]

A.4 Allison and McEwen, 2000

Allison00=[1] use the Mars pole vector of Folkner97=[2] and "Mars heliocentric coordinates, as referred to the ecliptic and mean equinox of date, were calculated from two different truncations of the high-precision VSOP87 representation of planetary orbits"; with "a maximum error of 0°.001". The time system used is Terrestrial Time (TT) from J2000.0 in days, $\Delta t_{\mathrm{J2000}}$, and τ is in years. The equation numbers in brackets are from the Allison article.

$t \equiv$ JD^{TT} -2451545. = MJD + (TT-TDB)   [15]

$$M = 19.3870 + 0.52402075t \qquad [16]$$

$$\alpha = afms = 270.3863 + 0.52403840t \qquad [17]$$

$$PBS = \sum_{i-1} A_i \cos\left[.985626^\circ t/\tau_1 + \phi_i\right] \qquad [18]$$

the constant factor is 360/365.25.

$$B = \left(10.691 \underbrace{+3. \times 10^{-7} t}_{\text{tiny}} \right) \sin M + \sum_{i=2}^{5} b_i \sin iM \qquad [19a]$$

$$L_S = \alpha + B + PBS \qquad [19b]$$

ignoring "tiny", this reduces to

$$L_S = c_1 + c_2 t \quad + \sum_{i=1}^{5} b_i \sin iM \quad + PBS$$

Allison00=[1] Table 5 lists seven short-term periodic perturbations due to the other planets, the largest amplitude is 0.007°and the sum of all is 0.027°

I coded this in IDL as **LSAM** with input time in UTC and developed the reverse function (L_S to date) with an empirical addition to minimize closure.

The difference of **LSAM** from the L_S points here: mean=0.0038, StdDev=0.008 , min=-0.012, max=0.025

A.5 Generic (any body) L_S algorithm

My IDL routine **lsubsgen.pro** will generate L_S at any time for a body for which the Keplerian orbital elements (and the central mass) and spin-axis direction are know.

Do once: Using body orbital elements and spin axis in the same coordinate system; compute direction of the VE, the angle from the VE to periapsis, the time of periapsis, and the rotation matrix from the F system (Focal: +Z toward right-hand orbit normal, +X toward periapsis) to the B system (Body: +Z toward right-hand spin axis, +X toward bodies rising node of its orbit which is also the anti Vernal Equinox)

Each time: Convert time to mean anomaly, calculate the true anomaly, get the HP vector in the F system, rotate it into the B system to get the sub-solar declination; the magnitude of the vector is the heliocentric distance. Offset the true anomaly to get L_S.

My IDL routine LSUBSGEN does this in two steps:
Kode=0: Given the Keplerian elements (and the central body gravitational constant), compute a set of intermediate constants
Kode=1: for a single or vector of request dates (MJD, relative to J2000.0):
    Compute the mean anomaly (radians)
    Subtract the true anomaly of the anti Vernal Equinox and convert to degrees.

A.6 LsHK, derived here

The three analytic forms derived here have been coded into the IDL routine **lshk,pro**; with the keyword `hi`: 0 yields low precision (0.05 deg) relations, 1 yields high-precision (0.002 deg) version, and 2 yields the higher-precision (0.001 deg) version. With hi=2, the difference of **LsHK** from the L_S points here: mean=2.e-6, StdDev=0.0012 , min=-0.0046, max=0.0042; all units are degrees.

B Main program parameters that can be modified

```
@11 File names: parf
  0 Standish               = ~/krc/porb/standish.tab
  1 horizons               = ~/krc/porb/horizonsOutput
  2 Output Dir             = ~/mars/MYcalendar/
  3  file for full table   = Ls0
  4  " Pub1                = LsCal
  5 VernalEq code >>       = V33
  6 @4   MY=mba    >>    \ = mba
  7 suffix for 6:11   >> | = 36U
  8 @441 Positions=mxb3   | = mbx
  9 @445 oPole=mopp       | = opole
```

```
 10 @445 pole=pfit [.bin5]| = pfit
 11 @443 xxyy           /  = xxx
 12 Horizons DIR           = /work1/themis/
 13  " file                = h2y_mars
 14  " .ext                = .txt
 15 PCP table[.tab]        = pcp1

@12 Integer values: pari
       0       4  Numtimes
       1       3  Column sets per page
       2       1  0=ecliptic 1=Equitorial
       3      10  @4 jmax
       4    -184   " my1
       5     285   " Num MY
       6       2  Marspin kode
       7       2  Spinup years
       8       3  @4 jmin
       9      50   " " lines/page
      10       0  Offset for PRINTAB53
      11       5  LaTeX line mod
      12      -1   " " offset
      13       0   " all flag
      14      90  -10*log_10 tolerance
      15      36   N/year @44 Avoid odd
      16       4  Option for orbit Normal
      17       1  @4 flag for UniformYears
      18     800  Num plot for FFT
      19       0  Flag: use MJD in table

@13 Integer values: parj
       0    1600  spin fit: year1
       1    2200   " " last
       2    7777   " Num times
       3       2  fit degree
       4     444  seed, 0=time
       5       3  Fitsine: degree
       6      90   " maxIteration
       7      45   " -10*log10 tolerance
       8       0  FitCos: degree
       9      40   " -10*log10 adequate MAR
      10       2  Number basic terms
      11       0  Number PBS terms
      12       0  Flag to use normal equations
      13       0  Flag: LS=0 based on Z
      14       1  Flag: year length is a parameter
      15       1  Num eccen terms, 0=sine series

@14 Fit ranges: parp
       0      0.00000  FitAnom Phase; initial
       1      1.00000   " offset
       2     -40.0000   " minimum
       3      40.0000   " maximum
       4     -5.00000  exp 10 factor for next four
       5      0.00000  FitAnom rate; initial
       6   -0.0100000   " offset
       7     -1.00000   " minimum
       8      1.00000   " maximum
       9      1.00000  PBS offset
      10      85.0000   " =/- limit, degree
```